PACM$\wedge$N 2026
PROGRAMME & ABSTRACTS

VERONA, 10–12 JUNE 2026

UNIVERSITÀ DEGLI STUDI DI VERONA
POLO UNIVERSITARIO E MUSEALE "SANTA MARTA"
VIA CANTARANE, 24 – VERONA

PRELIMINARY TIMETABLE

Wednesday 10		Thursday 11		Friday 12	
		9:30	Longo	9:30	Rosolini
10:15	Herbelin	10:15	Primiero	10:15	Misselbeck-Wessel
11:00	– *coffee* –	11:00	– *coffee* –	11:00	– *coffee* –
11:30	Kamide	11:30	Piazza	11:30	Berardi
12:15	Uthaiwat	12:15	Mosca	12:15	D'Asaro
12:45	– *lunch* –	12:45	– *lunch* –	12:45	– *lunch* –
14:45	Laurent	14:45	Pascucci	14:45	Borsetto
				15:15	Carlucci
15:30	Palazzo	15:30	Bilotta		
16:00	– *coffee* –	16:00	– *coffee* –	16:00	– *closure* –
16:30	Guerrieri	16:30	Faggian		
17:15	Benini	17:15	Giuntini		
18:00	– *closure* –	18:00	– *closure* –		
		20:00	– *social dinner* –		

Under the auspices of

Myths, Legends, Tested Truths.

Marco Benini (Università degli Studi dell'Insubria).

Abstract. The strong normalisation (SN) theorem is a cornerstone property of type theories. In the context of Homotopy Type Theory (HoTT), the underlying Martin-Löf type theory (which we denote as MLU) features a cumulative hierarchy of universes. It is widely believed—and explicitly claimed in the foundational 2013 HoTT book—that strong normalisation holds for this system. However, a careful examination reveals this claim to be a mathematical myth.

This talk traces the history of this misconception. We show that canonical SN proofs, such as those by Martin-Löf (1973) and Abel, Coquand, and Dybjer (2007), rely fundamentally on non-cumulative universes or unique typing up to convertibility. These proofs fail to extend to MLU, where a single term can inhabit non-equivalent types. Furthermore, we demonstrate that applying standard η-contraction in this setting actively breaks subject reduction.

To transition from legend to "tested truth," we present a novel, rigorous proof of the strong normalisation and Church-Rosser theorems for MLU. We resolve the subject reduction failure by introducing a strictly type-aware definition of η-contraction, and we establish the necessary structural foundations by assuming the injectivity of the Π constructor. By rewriting the theory using an elimination-oriented syntax and employing the inductive techniques of Joachimski and Matthes (2003), we successfully recover the SN property. Finally, we conclude by highlighting a significant open question: whether Π-injectivity must be externally imposed, or if it can ultimately be proved internally within MLU.

Higman Lemma in Constructive Mathematics.

Stefano Berardi (Università degli Studi di Torino) — Joint work with Gabriele Buriola and Peter Schuster.

Abstract. Higman's lemma says that if the letters of a (possibly infinite) alphabet are well quasi-ordered, then the finite words are well quasi-ordered by the induced embedding order. We redefine the notion of well quasi-order by way of bars and give a constructive proof, with intuitionistic logic, of the corresponding Higman lemma for bars whenever the given quasi-order on letters is a decidable relation, i.e. any two letters are either related or else they are not.

Growing HOLMS: A Modular Framework for Modal Logics within HOL Light Proof Assistant.

Antonella Bilotta (Scuola Normale Superiore, Pisa) — Joint work with Marco Maggesi, University of Florence, and Cosimo Perini Brogi, IMT Lucca.

Abstract. This talk introduces HOLMS (HOL Light Library for Modal Systems), a framework designed for formalising and implementing a variety of modal logics. We present the latest developments of the library, which currently provides uniform formalisations—via axiomatic calculi, semantics, and labelled sequent calculi—alongside certified automated theorem proving for nine normal modal systems. Seven of these logics ($\mathsf{K}, \mathsf{D}, \mathsf{T}, \mathsf{B}, \mathsf{K4}, \mathsf{S4}$, and $\mathsf{S5}$) belong to the modal cube and are implemented using a highly modular strategy. The library also covers Gödel-Löb logic (GL) and Grzegorczyk logic (Grz). For the latter, we explore a novel implementation strategy via modal translation: by embedding Grz into GL, we leverage the existing mechanisation of GL to provide automated support for Grzegorczyk logic.

Kraus's Magic Trick for Computing with Constructive Reals.

Riccardo Borsetto (Università degli Studi di Verona) — Joint work with Lorenzo Molena and Marcin Jan Turek-Grzybowski.

Abstract. In constructive mathematics, a real number can be an abstract object, fixed by the axioms of a Cauchy-complete Archimedean ordered field, or something computed through a chosen representation, rarely both. In Cubical Type Theory, higher-inductive Cauchy reals keep the abstraction while recovering some computation: they are built as an internal completion of the rationals, with the required identifications built into the type itself, so equality is the identity type. The approximation result is therefore merely existential: for each real and positive precision, some rational approximation exists, but no uniform operation extracts it, since at fixed precision this would amount to a nonconstant, discontinuous observation of arbitrary reals. Kraus's "magic trick", realised in Cubical Agda, is a general normalisation principle for types where every element can be reached from a chosen one by an automorphism. For the rationals, decidable equality provides such automorphisms, so a hidden rational witness reduces definitionally to that rational. In this way, computationally defined closed real terms yield concrete digits without postulating a digit-extraction function for arbitrary reals, or assuming excluded middle or choice.

Partition theorems, barriers and well-ordering principles: computability-theoretic and proof-theoretical aspects.

Lorenzo Carlucci (Università degli Studi di Roma "La Sapienza").

Abstract. The logico-computational strength of Ramsey-type partition theorems is of central interest in reverse mathematics and computable combinatorics. In this talk I give an overview of results concerning generalizations of Ramsey's theorem and of some of its consequences to "relatively large sets" in the sense of Paris-Harrington and to "barriers" in the sense of Nash-Williams, with particular attention to the relations with well-ordering preservation principles. The focus is on schemes of proof and on the interplay between computability-theoretic reductions and proof-theoretic implications.

Non-monotonic Depth-Bounded Reasoning.

Fabio Aurelio D'Asaro (Università degli Studi di Verona) — Joint work with Paolo Baldi, University of Salento.

Abstract. Agents with commonsense diverge from classical logic in two opposite directions: (i) they infer more than classical logic licenses, "jumping to conclusions" that may be retracted when new information arrives, and (ii) they infer less, since classical inference is computationally hard and real agents reason under limited time, attention, memory, and computation. While (i) is captured by non-monotonic logic, a proposal for (ii) is given by Depth-Bounded Boolean Logics (DBBLs), a family of tractable systems approximating classical logic (D'Agostino et al., 2024).

We propose a formal framework combining these two strands, integrating DBBLs with the answer-set semantics of logic programming. A central observation is that change in reasoning arises not only from new information, but also as further inferential resources become available: shallow reasoning relies on quick, revisable inferences, while deeper reasoning inspects alternatives, derives stronger conclusions, and retracts some premature commitments. Increasing inferential depth thus works against non-monotonicity. Our main contribution is a hierarchy of depth-bounded non-monotonic consequence relations, modeling agents with increasing depth and decreasing non-monotonicity, with classical logic recovered on the limit. We also provide an ASP-based implementation that builds upon k-lingo (D'Asaro, Baldi & Primiero, 2021), and finally discuss whether deep reasoning may enable low-depth agents to learn heuristics for clusters of similar problems.

The proof-theory of Bayesian inference: proof-nets, Bayesian networks, and quantum Bayesian networks.

Claudia Faggian (Chargée de Recherche, CNRS, Laboratoire IRIF, Université Paris Cité) — Joint work with Rémi di Guardia and Thomas Ehrhard.

Abstract. Bayesian networks are a prominent graphical formalism for probabilistic reasoning, enabling compact factorized representations and efficient inference algorithms.

This talk presents a proof-theoretic view of Bayesian reasoning through the graphical syntax of linear logic, namely proof-nets. In the spirit of Curry-Howard, we show how cut-elimination, and its dual cut-expansion, correspond to factorized inference algorithms. Bringing Bayesian networks into the framework of linear-logic proof theory, which is intrinsically resource-aware, accommodates not only compositional graphical reasoning and a typing discipline for modularity, but also efficient computation.

We then extend this perspective to *quantum* Bayesian networks. The main challenge is to reconcile the sharing of classical data with the quantum no-cloning principle.

From Classical Logic to Fuzzy Quantum Logic.

Roberto Giuntini (Università degli Studi di Cagliari).

Abstract. The unsharp approach to quantum mechanics (QM) [2] is rooted in a foundational question of Hilbert-space quantum theory. Given an event–state system $(\Pi(\mathcal{H}), \mathcal{S}(\mathcal{H}))$ — with $\Pi(\mathcal{H})$ the set of projections and $\mathcal{S}(\mathcal{H})$ the set of density operators of the Hilbert space $\mathcal{H}$ — do these furnish the optimal mathematical counterparts of the intuitive notions of *event* and *state*? Once $\Pi(\mathcal{H})$ is fixed, Gleason's Theorem guarantees that $\mathcal{S}(\mathcal{H})$ is optimal as a notion of state: whenever $\dim \mathcal{H} \geq 3$, every probability measure on $\Pi(\mathcal{H})$ is determined by a density operator. The set of events, by contrast, is *not* maximal: there exist bounded linear operators E that are not projections yet still satisfy Born's rule, i.e. $\mathrm{Tr}(\rho E) \in [0,1]$ for every density operator ρ.

The unsharp approach therefore liberalizes the notion of quantum event, replacing $\Pi(\mathcal{H})$ by the set $\mathcal{E}(\mathcal{H})$ of all *effects* — the bounded linear operators E with $\mathbb{O} \preceq E \preceq \mathbb{I}$ — so that $\Pi(\mathcal{H}) \subsetneq \mathcal{E}(\mathcal{H})$. In a nutshell, effects stand to projections as the unit interval $[0,1]$ stands to $\{0,1\}$: the sharp, bivalent events of $\Pi(\mathcal{H})$ give way to the unsharp, fuzzy events of $\mathcal{E}(\mathcal{H})$.

The set $\mathcal{E}(\mathcal{H})$ carries a natural structure [1, 2] of *Brouwer–Zadeh poset* (BZ-poset) $\langle \mathcal{E}(\mathcal{H}), \leq, ', ^\sim, \mathbb{O}, \mathbb{I} \rangle$, endowed with two complements: a fuzzy-like (Kleene) complement $E' = \mathbb{I} - E$ and an intuitionistic-like (Brouwer) complement $E^\sim = P_{\mathrm{Ker}(E)}$, the projection onto the kernel of E. This BZ-poset is *properly fuzzy*: the noncontradiction principle fails, since $E \wedge E' \neq \mathbb{O}$, and $\mathcal{E}(\mathcal{H})$ is not even a lattice [2]. In a rather neglected paper, however, Olson [4] showed that the *spectral order* $\leq_s$ turns $\mathcal{E}(\mathcal{H})$ into a complete lattice, in which the projections are precisely the *sharp* effects, those satisfying $E \vee E' = \mathbb{I}$.

We investigate the algebraic structure $\langle \mathcal{E}(\mathcal{H}), \leq_s, ', ^\sim, \mathbb{O}, \mathbb{I} \rangle$. With respect to $\leq_s$ it is a Kleene lattice that is no longer orthomodular — orthomodularity would reinstate the excluded middle — yet it is still *paraorthomodular*. This leads us to the variety of *PBZ*-lattices* (paraorthomodular BZ*-lattices), of which $\mathcal{E}(\mathcal{H})$ is the standard model and which provide a faithful abstraction of it [3, 5, 6]. In this setting the several *a priori* distinct notions of "unsharpness" (Kleene-, Brouwer- and $\Diamond$-sharpness) collapse into a single one, exactly as they do for the concrete effects. We finally develop the structure theory of PBZ*-lattices — showing, in particular, that the variety generated by the concrete lattices of effects is a *proper* subvariety — and give a first description of the lattice of PBZ*-varieties.

[1] G. Cattaneo and G. Nisticò, *Brouwer–Zadeh posets and three-valued Łukasiewicz posets*, Fuzzy Sets and Systems **33** (1986), 165–190.

[2] M. L. Dalla Chiara, R. Giuntini, and R. Greechie, *Reasoning in Quantum Theory*, Kluwer, Dordrecht, 2004.

[3] H. F. de Groote, *On a canonical lattice structure on the effect algebra of a von Neumann algebra*, arXiv:math-ph/0410018v2 (2005).

[4] M. P. Olson, *The selfadjoint operators of a von Neumann algebra form a conditionally complete lattice*, Proceedings of the American Mathematical Society **28** (1971), 537–544.

[5] R. Giuntini, A. Ledda, and F. Paoli, *A new view of effects in a Hilbert space*, Studia Logica **104** (2016), 1145–1177.

[6] R. Giuntini, A. Ledda, and F. Paoli, *On some properties of PBZ*-lattices*, International Journal of Theoretical Physics **56** (2017), 3895–3911.

The Theory of Meaningfulness in Untyped lambda-Calculi.
Giulio Guerrieri (University of Sussex) — This contribution collects some results that the author obtained in collaboration with Alberto Carraro, Simona Ronchi Della Rocca, Luca Paolini, Beniamino Accattoli, Giulio Manzonetto, Delia Kesner, Victor Arrial, and others.

Abstract. One of the main goals of programming language theory is the rigorous mathematical logic study of the *meaning* of programming languages. The *untyped λ-calculus* [15] is a simple and Turing-complete model of computation that represents the kernel of any functional programming language. There are actually many variants of the λ-calculus, mimicking different parameter passing styles in programming languages such as *call-by-name* and *call-by-value*, each one with its own theory and notion of meaningfulness. Via the Curry-Howard correspondence between programs and proofs, Girard's linear logic [14] inspired the introduction of variants of the λ-calculus, such us the *bang calculus* [7, 8], subsuming both call-by-name and call-by-value thanks to Girard's two translations of intuitionistic logic into linear logic.

The study of the semantics of the untyped *call-by-name* λ-calculus (CbN) is a well-developed field built around the concept of *solvable* terms, which are elegantly characterized in many different ways [12, 11, 13, 10]. In particular, unsolvable terms provide a *consistent* notion of meaningless terms (in that they can all be equated without collapsing the induced equational theory), and meaningful terms can be identified with the solvable ones.

The study of the semantics of the untyped *call-by-value* λ-calculus (CbV, which is closer to the real implementations of programming languages [9]) is instead still in its early stages, because of some inherent difficulties but also because CbV solvable terms are less studied and understood than in CbN. On the one hand, we show that a carefully crafted presentation of CbV [5, 4], inspired by linear logic, allows us to recover many of the properties that solvability has in CbN, in particular logical (via multi types) and operational (via a special reduction) characterizations. On the other hand, we stress that, in CbV, solvability plays a different role: identifying unsolvable terms as meaningless induces an inconsistent equational theory actually identifying all terms [1].

We argue that in CbV, the correct notion of meaningful terms is captured by the concept of *potential valuability* [6]. In particular, terms that are not potentially valuable enjoy the same good properties as unsolvable terms in CbN:

(1) they can be characterized logically and semantically via multi types [1];
(2) they can be characterized operationally as non-termination of a special reduction [5];
(3) they fulfill the *genericity lemma* according to which a term that is not potentially valuable, plugged into a term whose evaluation terminates, plays no role in that evaluation [2];
(4) they provide a *consistent* notion of meaningless terms in CbV (in that they can all be equated without collapsing the induced equational theory) [2].

Moreover, we can subsume the notions of meaningfulness for both CbN and CbV in a general notion of meaningfulness for the *bang calculus* [3]. Finally, we show that the study of meaningfulness in the bang calculus sounds more intricate than in CbN and CbV λ-calculi, because the bang calculus is a more expressive language.

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[2] Arrial, V., Guerrieri, G. & Kesner, D. Genericity Through Stratification. *Proceedings Of The 39th Annual ACM/IEEE Symposium On Logic In Computer Science, LICS 2024*. pp. 5:1-5:15 (2024), `https://doi.org/10.1145/3661814.3662113`
[3] Kesner, D., Arrial, V. & Guerrieri, G. Meaningfulness and Genericity in a Subsuming Framework (Invited Talk). *9th International Conference On Formal Structures For Computation And Deduction, FSCD 2024*. pp. 1:1-1:24 (2024), `https://doi.org/10.4230/LIPIcs.FSCD.2024.1`
[4] Carraro, A. & Guerrieri, G. A Semantical and Operational Account of Call-by-Value Solvability. *Foundations Of Software Science And Computation Structures - 17th International Conference, FOSSACS 2014, Proceedings*. pp. 103-118 (2014), `https://doi.org/10.1007/978-3-642-54830-7_7`
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[11] Barendregt, H. Solvability in lambda-calculi. *Colloque International De Logique : Clermont-Ferrand, 18-25 Juillet 1975.* pp. 209-219 (1977)

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[15] Church, A. The Calculi of Lambda Conversion. (Princeton University Press,1941), http://www.jstor.org/stable/j.ctt1b9x12d

Realising subsystems of second-order arithmetic with a separation-based variant of System F.

Hugo Herbelin (INRIA – Institut national de recherche en informatique et en automatique) — Joint work with Firmin Martin.

Abstract. System T is the language of realisers canonically associated to first-order Arithmetic while System F is the language of realiser canonically associated to second-order arithmetic. The languages of realisers canonically associated to the "big five" subsystems of second-order arithmetic should then naturally appear as restrictions of System F. Two of the big five, namely WKL? and ATR?, are however defined not by a comprehension axiom but by a separation axiom (or, equivalently, an interpolation axiom). In turn, comprehension corresponds in System F to substitution of second-order variables. To characterise versions of System F canonically associated to the big five, we then introduce a variant of System F where substitution of second-order variables is weakened so that it interprets instead separation (or interpolation).

Unified Proof Theory for Intermediate and Ecumenical Logics.

Norihiro Kamide (Nagoya City University) — Joint work with Sara Negri.

Abstract. The first section of this talk is devoted to unified proof theory for intermediate logics. G0- and G3-style sequent calculi are defined for the propositional intermediate logics Dummett logic and Jankov logic. The equivalence between the G0- and G3-style sequent calculi for these logics is then proved, from which cut elimination for these calculi follows. Natural deduction systems with general elimination rules are also defined for these logics, and full normalization theorems for these systems are established. These normalization results are obtained by means of bidirectional translations between the G0-style sequent calculi and the natural deduction systems. Additionally, alternative cut-free sequent calculi and a fully normal natural deduction system for classical logic are obtained by generalizing the proposed proof systems. The second section of this talk is devoted to unified proof theory for ecumenical logic, which combines classical logic and intuitionistic logic within a single framework, allowing them to coexist and interact. In this section, a new ecumenical logic is introduced. Results for sequent calculi and natural deduction systems similar to those discussed above for the intermediate logics are obtained for the proposed ecumenical logic.

What if true = false? (in a linear setting).

Olivier Laurent (Directeur de Recherche at CNRS, Laboratoire de l'Informatique du Parallélisme (LIP), ENS Lyon).

Abstract. To make the question "What if true = false?" meaningful, one needs to specify what is the surrounding logic, what is true, what is false and which equality between them is considered. While true = false immediately makes classical logic degenerated (all formulas are provable), we discuss how it is sensible to distinguish two notions 0 and $\bot$ for false in intuitionistic logic so that adding true = $\bot$ (as opposed to true = 0) does not make the logic completely degenerated. Then we move to the linear logic setting where these two notions of false exist as well as two notions $\top$ and 1 of true. Again $\top = 0$ or $\top = \bot$ or $1 = 0$ makes all formulas provable. A last case $1 = \bot$ remains, which is known to be related with the mix rules of linear logic. Digging into the computational content of proofs, we refine the analysis with respect to three levels of "equality" for $1 = \bot$: equivalence $1 \dashv\vdash \bot$, isomorphism $1 \simeq \bot$ and identity $1 \equiv \bot$. After showing that, in this particular case, the three notions almost coincide, we will present syntax and semantics of MLLM: multiplicative linear logic with a self dual unit $I \equiv I^{\bot} \equiv 1 \equiv \bot$.

The difference between generalization and conceptual transfer: Setting Limits, from Mathematics to Biology.

Giuseppe Longo (CNRS – Centre national de la recherche scientifique).

Abstract. We will single out some fundamental invariants of the mathematics of computability and extend them to a broader structural understanding. How to generalize finiteness and definability? Which is the invariant mathematical character of self-application? As G. Kreisel observes, "What one has to guard against is to imitate mechanically the basic developments of recursion theory". We will compare this vision of mathematics and its application to physics, where the search and the proposal of fundamental mathematical invariants becomes a key tools for knowledge, and then move to the abuses and the flat transfer of basic notion of computability, as such, onto other forms of knowledge. We will in particular briefly hint to the physico-mathematical and the bio-mathematical inconsistencies of claims concerning the physical universe as a "computing machinery" and the biological dynamics as "programmed processes" and stress the relevance of Negative Results, as Poincaré called his classical Three Body theorem.

[1] A. Nocek (with G. Longo) The Organism Is a Theory. Giuseppe Longo on Biology, Mathematics, and AI, University of Minnesota Press, Minneapolis, London, 2026.
— Nocek-on Longo's-work-Excerpts.pdf — New Books podcast

Richman rigs revisited.

Daniel Misselbeck-Wessel (Munich Center for Mathematical Philosophy (MCMP), Ludwig-Maximilians-Universität München) — Based on joint work with Iosif Petrakis.

Abstract. Along with his treatment of algebraic real functions [1], Richman pursued the question of an axiomatic approach to algebras of partial maps. His proposal aimed at certain semirings in which the zero is not required to be multiplicatively absorbing, and where a "somewhat strange looking rule" allows for the additive idempotents to track domains.

In this talk, we revisit Richman's proposal, close a gap in his original setting, and provide a simple equational base for the variety of his intended semirings, which we call *Richman rigs.* A refined version of Richman's structure theorem then follows from the theory of Płonka sums.

In the multiplicatively idempotent case, we obtain a representation theorem in terms of complemented set algebras, and we develop a corresponding Stone-type duality by means of *layered spaces.*

[1] Fred Richman. Algebraic functions, calculus style. *Comm. Algebra*, 40(7):2671–2683, 2012.

Algorithms and Learning-Based Programs.

Caterina Mosca (LIPN – Laboratoire d'Informatique de Paris-Nord).

Abstract. The term *algorithm* has a long history that predates computer science by many centuries. Originally used to describe mathematical procedures, its meaning became progressively specialised in a computational sense and becoming the privileged object of study of computer science. More recently, with the rise of AI systems, the term is taking on an yet another meaning, denoting what computer scientists and mathematicians call *models*. This talk is motivated by the natural question raised by this shift in usage: how does this emerging notion of an "algorithm" relate to the more traditional one? I will first argue that, in a certain sense, most programs—and therefore most programs obtained through training—do not admit an algorithmic description. This observation will then lead me to explore a number of technical questions concerning neural networks viewed as a (limited) model of computation, and whose answers could shed light on the situation. Can one define a notion of *convergence to an algorithm*? Can the efficiency of a trained program reveal structural properties of the task for which it was trained?

Towards a Reversible Characterization of L.

Matteo Palazzo (Università degli Studi di Torino).

Abstract. Reversible computational models are deterministic in both forward and backward execution. Originally introduced as a response to the "Maxwell's demon paradox", they show that information erasure necessarily entails energy dissipation. Among their many applications, reversible models have attracted increasing interest in complexity theory, particularly for the study and characterization of complexity classes..

We introduce $\mathsf{SRL}^{\%}$, a reversible language that implicitly characterizes RL, the class of problems solvable in logarithmic space by reversible Turing machines. Since $\mathsf{RL} = \mathsf{L}$ (a non-obvious result by Lange et al. from 2000), $\mathsf{SRL}^{\%}$ also characterizes L.

$\mathsf{SRL}^{\%}$ contributes to a broader line of research aimed at characterizing complexity classes through programming languages, recasting results from computability and complexity theory within a programming-oriented framework. This approach may lead to more intuitive formulations of definitions, theorems, and proofs, as well as to new technical insights and stronger results.

Formalizing the empirical analysis of sufficient and necessary conditions.

Matteo Pascucci (Central European University).

Abstract. In everyday reasoning, sufficient conditions and necessary conditions are often defined in terms of empirical tests that involve both defeasible and indefeasible arguments. One starts with a dataset of observations D, indicating correlations between a set of candidate conditions CC and a set of target conditions TC. The claim that A (CC) is a sufficient condition for B (TC) constitutes the conclusion of a defeasible argument based on the fulfilment of certain criteria in D, whereas the claim that A (CC) is not a sufficient condition for B (TC) is the conclusion of an indefeasible argument based on the violation of some criterion in D. The dataset of observations is typically extended over time and sometimes we find ourselves in a situation of uncertainty. In this talk, I will present a multimodal logic that allows one to capture such an empirical and fine-grained analysis of sufficient and necessary conditions.

An intrinsic additive quotient for classical propositional logic.

Mario Piazza (Scuola Normale Superiore, Pisa).

Abstract. We isolate a canonical additive quotient associated with classical propositional logic. The construction starts from canonical proof decomposition in fractional semantics: each formula A has an evidence profile $\mathsf{Ev}(A) = (\mathsf{Id}(A), \mathsf{Def}(A))$, where $\mathsf{Id}(A)$ and $\mathsf{Def}(A)$ count, respectively, identity and complementary top sequents in its canonical decomposition. Equality of evidence profiles induces an equivalence relation on formulas, and conjunction descends to the quotient $\mathsf{Fm}/{\equiv_{\mathsf{Ev}}}$, yielding a commutative semigroup.

We prove that this semigroup is the free commutative semigroup on two generators; after adjoining a zero element, its completion is isomorphic to $(\mathbb{N}^2, +)$. It follows that every nonzero evidence profile is realized by a classical formula, every formula admits a canonical evidence normal form, and the first-order theory of the completed quotient reduces effectively to Presburger arithmetic, with an explicit completeness theorem for its universal affine fragment.

The usual fractional value is recovered by normalizing the evidence profile. This quotient is distinct from the Lindenbaum–Tarski Boolean quotient: it records decomposition-sensitive data that disappear under truth-equivalence. The defect coordinate induces a discrete Lawvere cost semantics and the associated Lawvere quasi-metric geometry.

A Proof System with Causal Labels.

Giuseppe Primiero (Università degli Studi di Milano Statale) — Joint work with Leonardo Ceragioli.

Abstract. Ensuring fairness in machine learning systems requires identifying and formally reasoning about essential resources such as probabilistic dependencies, causal relations, and structural constraints. In this work, we present the typed natural deduction calculus TNDPQ which integrates causal labels directly into proofs, enabling principled reasoning about how protected attributes influence probabilistic outcomes in classifiers. We characterise individual fairness and intersectionality through structural constraints on inference rules and the analysis of conditional independence. The calculus is extended to model counterfactual reasoning by representing interventions and hypothetical scenarios over causal structures. The system provides a sound and expressive post-hoc method for analysing fairness in black-box classifiers, bridging logical proof systems, causal reasoning, and contemporary fairness criteria in machine learning.

Cantor's Theorem in the light of Lawvere's Remarks.

Giuseppe Rosolini (Università degli Studi di Genova).

Abstract. In 1890, Cantor produced his famed theorem about the infinity of infinite cardinalities with the declared intention to give a new, very abstract proof of part of the 1874 theorem about the two cardinalities of the sets of natural numbers and of real numbers.

One proof is strikingly different from the other: the earlier result was a technical lemma to yield a result about transcendental numbers – a renowned theorem in the mathematical circles, though rarely associated with Cantor.

We present the two proofs in contemporary mathematical language, and show how Cantor's second approach applies directly to several other paradoxical situations.

But what is beta-reduction?

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Abstract. Through the lens of linear logic, we see that the untyped lambda calculus comes in two flavours which are called additive and multiplicative depending on whether the structural rules are explicit or admissible. The multiplicative presentation has the structure of a cartesian closed operad while the additive one has that of a closed clone. Beta and eta reductions play a role in defining both categorical structures. This insight is already present in the literature such as in Hyland's work which tells us that the untyped lambda calculus is the initial (semi)closed algebraic theory. Therefore, this talk will be centered around two points which explicit this approach and build on it:

1) The construction of a closed clone (which is one possible definition of an algebraic theory) from the (additive) untyped lambda calculus. To this end, we do a quick survey of multicategorical theory and of some of its contributions to denotational semantics (how to determine the multicategorical structure of a calculus, what is an interpretation categorically).

2) A two-dimensional extension of Hyland's approach in which the homsets of the clone become homcategories (of terms and their operational semantics). In this setting, beta and eta reductions are themselves data of the categorical structure (i.e. closed Cat-clone), contrarily to the situation before.